Paper ID: 1748

Interpretable Graph Signal Denoising Using Regularization by Denoising

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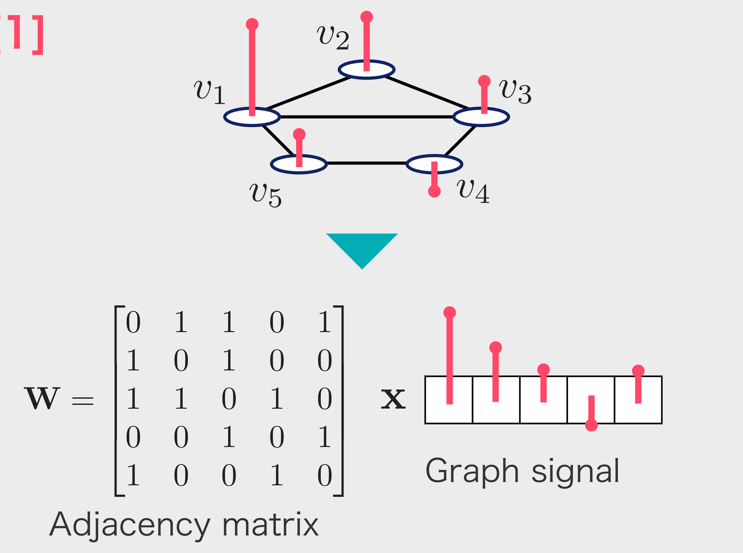
1. Introduction

Graph Signal Processing (GSP)[1]

- Signals often have their underlying structures.
- GSP can consider the underlying structure of signals.
  e.g. Transportation network, bioinformatics, 3D point cloud.



Regularization by Denoising (RED)[2]

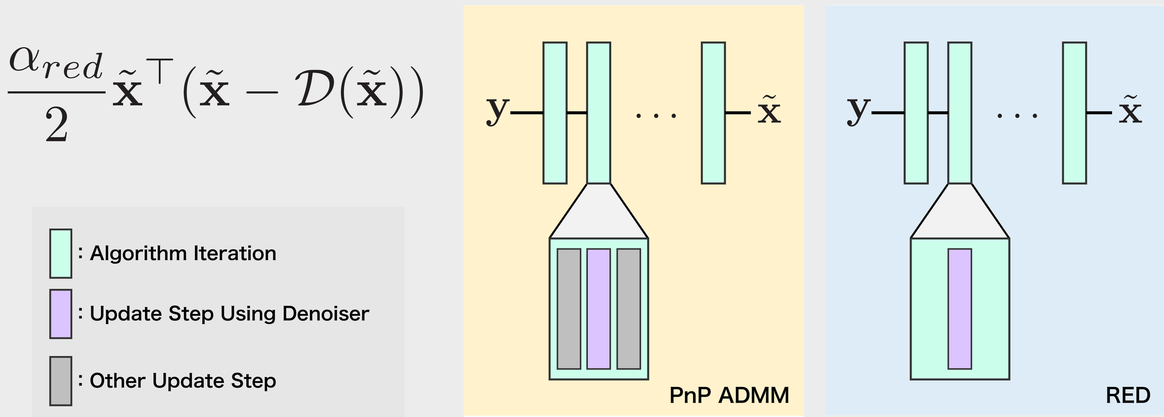
- RED is an image restoration method that uses image denoiser for its regularization term.
- In contrast to plug-and-play ADMM, RED is more interpretable because it directly use the denoiser as the regularization term.

$$\mathbf{x} = \underset{\tilde{\mathbf{x}}}{\operatorname{argmin}} \frac{1}{2}\|\tilde{\mathbf{x}} - \mathbf{y}\|^2 + \frac{\alpha_{red}}{2}\tilde{\mathbf{x}}^\top(\tilde{\mathbf{x}} - \mathcal{D}(\tilde{\mathbf{x}}))$$

$\tilde{\mathbf{x}}$: Denoised Signals

α_{red} : Hyperparameter

$\mathcal{D}(\cdot)$: Image Denoiser



- When $\mathcal{D}(\cdot)$ satisfies the following conditions, gradient of the regularization term can be written as:

$$\nabla\tilde{\mathbf{x}}^\top(\tilde{\mathbf{x}} - \mathcal{D}(\tilde{\mathbf{x}})) = \tilde{\mathbf{x}} - \mathcal{D}(\tilde{\mathbf{x}})$$ <u>without gradient of denoiser $\nabla\mathcal{D}(\cdot)$</u>

Condition 1: (Local) Homogeneity

$\mathcal{D}(c \cdot \tilde{\mathbf{x}}) = c \cdot \mathcal{D}(\tilde{\mathbf{x}})$ when c is very closed to 1.

Condition 2: Strong Passivity

Inner denoiser must be stable. $\eta(\nabla\mathcal{D}(\tilde{\mathbf{x}})) \leq 1$ $\eta(\cdot)$: Spectral radius

2. Proposed Method

Are Graph Signal Denoisers Applicable to RED?

① Laplacian Regularization for RED

$$\min_{\tilde{\mathbf{x}}} \|\tilde{\mathbf{x}} - \mathbf{y}\|^2 + \alpha_{lr}\tilde{\mathbf{x}}^\top\mathbf{L}\tilde{\mathbf{x}} \qquad \begin{aligned}\tilde{\mathbf{x}} &= f(\mathbf{L})\mathbf{y} \\ &= (\mathbf{I} + \alpha\mathbf{L})^{-1}\mathbf{y}\end{aligned}$$

$\mathbf{L}$: Graph Laplacian

Condition 1: (Local) Homogeneity

1) $\mathbf{L}$ is determined independently of the graph signal

$$f(\mathbf{L})(c \cdot \mathbf{x}) = c \cdot f(\mathbf{L})\mathbf{x}$$

2) $\mathbf{W}$ is inversely proportional to the distance between the signal values

$$[\mathbf{W}]_{i,j} = 1/\sqrt{\|[\mathbf{x}]_i - [\mathbf{x}]_j\|^2} \qquad [\mathbf{W}']_{i,j} = 1/\left(c\sqrt{\|[\mathbf{x}]_i - [\mathbf{x}]_j\|^2}\right)$$

if $\mathbf{W}$ is normalized, we always have local homogeneity.

Condition 2: Strong Passivity

if condition 1 is satisfied, $\nabla\mathcal{D}(\mathbf{x})\mathbf{x} = f(\mathbf{L})\mathbf{x}$. [2]

$$\eta(\nabla\mathcal{D}(\mathbf{x})) \leq 1 \qquad \eta(f(\mathbf{L})) \leq 1$$

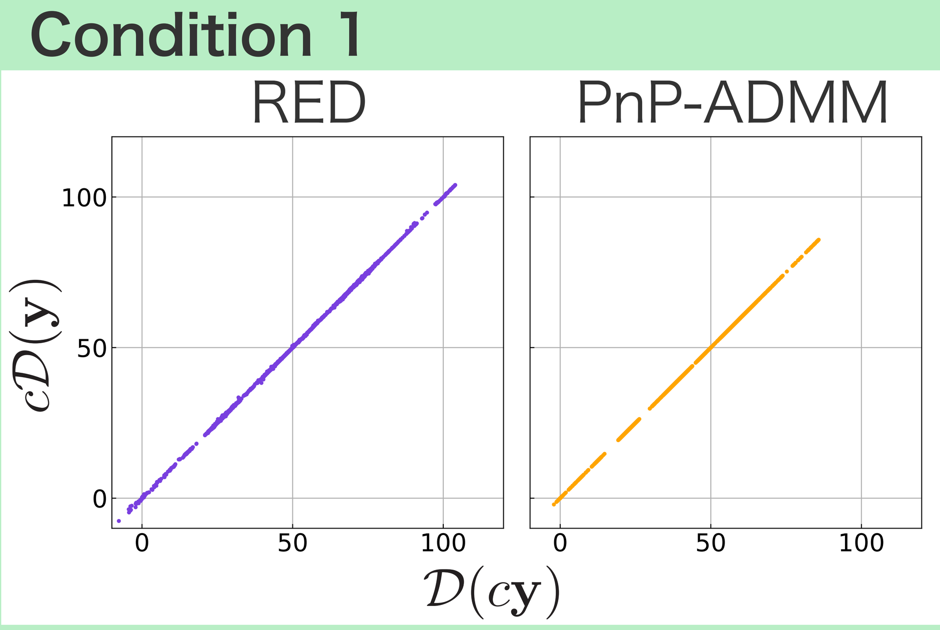
Since $\mathbf{L}$ is positive-semidefinite matrix,

$\eta(f(\mathbf{L})) = \eta((\mathbf{I} + \alpha\mathbf{L})^{-1}) \leq 1$.

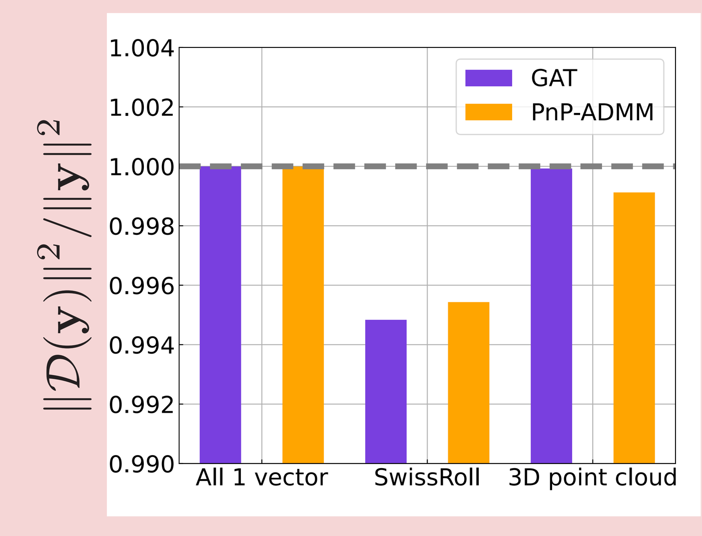
② Graph attention network and PnP ADMM for RED

We <u>experimentally</u> validate they satisfy the two conditions.

Condition 1



Condition 2



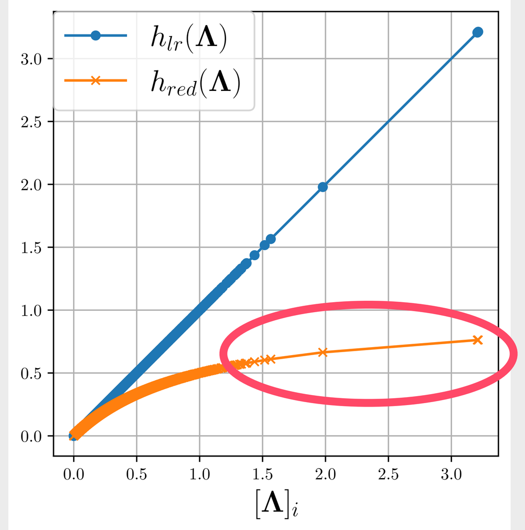
RED from Graph Filter Perspective

We compare gradient of regularization term.

- LR
$$\begin{aligned}\nabla\frac{\alpha_{lr}}{2}\mathbf{x}^\top\mathbf{L}\mathbf{x} &= \alpha_{lr}\mathbf{L}\mathbf{x} \\ &= \alpha_{lr}\mathbf{U}\boldsymbol{\Lambda}\mathbf{U}^\top\mathbf{x} \\ &= \alpha_{lr}\mathbf{U}h_{lr}(\boldsymbol{\Lambda})\mathbf{U}^\top\mathbf{x}\end{aligned}$$
- RED
$$\begin{aligned}\nabla\frac{\alpha_{red}}{2}\mathbf{x}^\top(\mathbf{x} - \mathcal{D}(\mathbf{x})) &= \alpha_{red}(\mathbf{x} - \mathcal{D}_{lr}(\mathbf{x})) \\ &= \alpha_{red}\left(\mathbf{x} - (\mathbf{I} + \alpha_{lr}\mathbf{L})^{-1}\mathbf{x}\right) \\ &= \alpha_{red}\mathbf{U}h_{red}(\boldsymbol{\Lambda})\mathbf{U}^\top\mathbf{x}\end{aligned}$$

$h_{lr}(\boldsymbol{\Lambda}) = \alpha_{lr}\boldsymbol{\Lambda}$

$h_{red}(\boldsymbol{\Lambda}) = \alpha_{red}h_{lr}(\boldsymbol{\Lambda})(\mathbf{I} + h_{lr}(\boldsymbol{\Lambda}))^{-1}$



RED passes high-frequency components compared to LR.

→ RED eliminates <u>oversmoothing</u>.

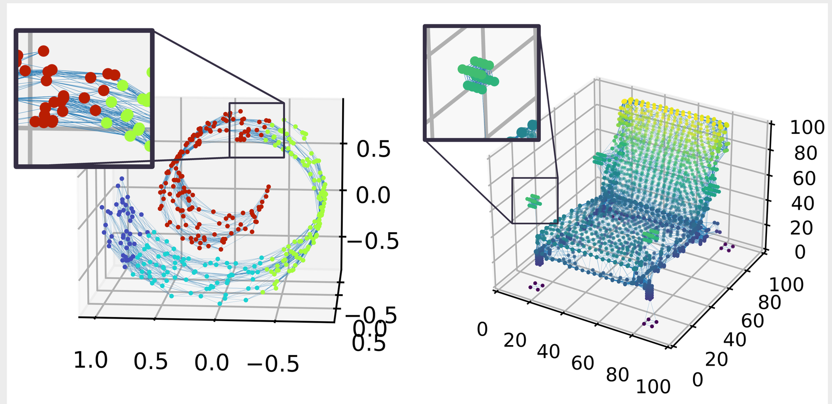
3. Experimental Results

Datasets

- Synthetic Dataset $N = 400$, k-NN ($k = 5$)
- ModelNet Variois N, k-NN ($k = 5$)

Evaluation Measure

$$RMSE(\mathbf{x}^*, \tilde{\mathbf{x}}) = \sqrt{\frac{1}{N}\|\mathbf{x}^* - \tilde{\mathbf{x}}\|_2^2}$$



Existing Methods

- Total variation (**TV**)[3]
- Laplacian Regularization (**LR**)[4]
- Graph Attention Network (**GAT**)[5]
- Graph signal denoising using **PnP-ADMM** (with LR)[6]

Denoising Performance

Synthetic dataset

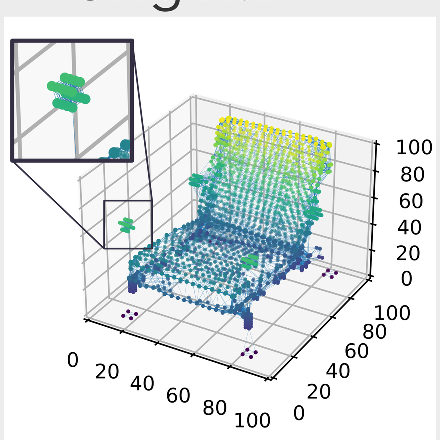
Noise Level σ^2	5	10	15	20	25	30
TV	2.24	3.18	3.79	4.42	4.93	5.57
LR	2.16	3.10	3.70	4.34	4.85	5.49
GAT	1.95	2.78	3.37	3.93	4.42	4.98
PnP-ADMM (LR)	1.78	2.65	3.22	3.82	4.32	4.96
Proposed (LR)	<u>1.69</u>	<u>2.31</u>	<u>2.75</u>	<u>3.18</u>	<u>3.61</u>	<u>4.09</u>
Proposed (GAT)	1.94	2.76	3.33	3.85	4.33	4.91
Proposed (PnP)	**1.63**	**2.05**	**2.39**	**2.72**	**3.13**	**3.54**

3D point cloud dataset

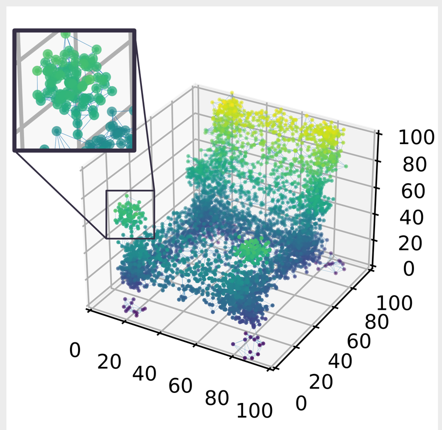
Noise Level σ^2	10	20	30	40	50	60
TV	3.23	4.36	5.54	6.38	7.09	7.70
LR	2.96	4.01	5.01	5.80	6.41	7.03
GAT	5.46	4.40	5.87	6.08	6.00	6.42
PnP-ADMM (LR)	**2.74**	3.51	<u>4.23</u>	4.79	5.22	5.80
Proposed (LR)	2.88	**3.44**	**4.09**	<u>4.46</u>	<u>4.82</u>	<u>5.37</u>
Proposed (GAT)	<u>2.77</u>	3.62	4.42	5.04	5.47	5.83
Proposed (PnP)	2.96	<u>3.45</u>	**4.09**	**4.39**	**4.73**	**5.27**

Visualization Results (3D point cloud)

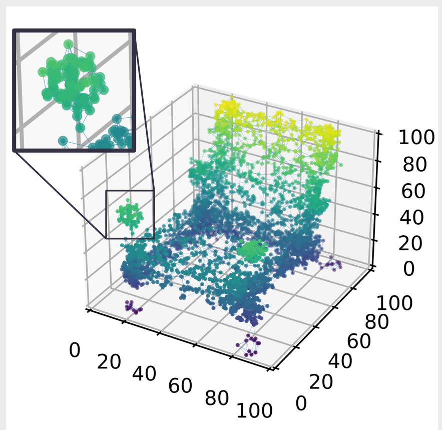
- Original



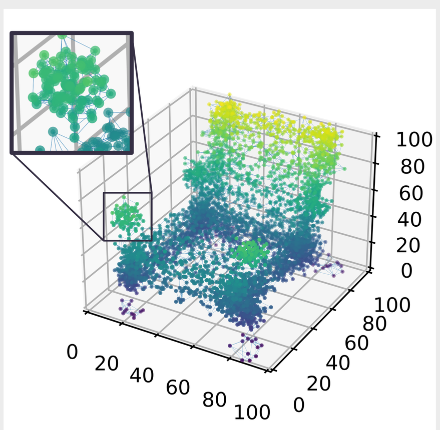
- LR



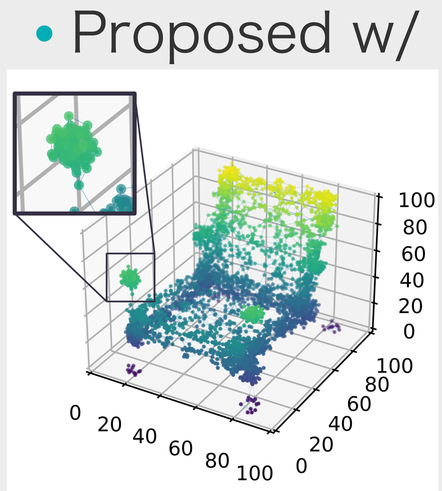
- PnP-ADMM



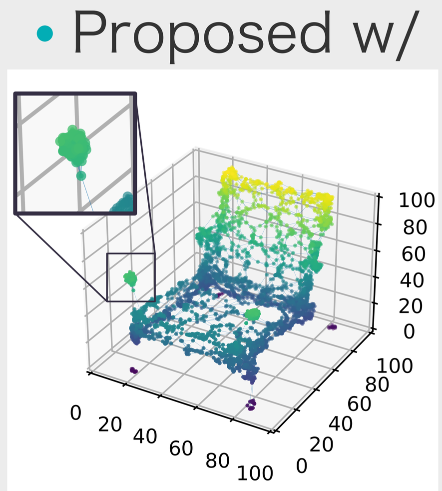
- Observed



- Proposed w/ LR



- Proposed w/ PnP



4. Conclusion

- This paper proposes a interpretable graph signal denoising method.
- Our proposed method is based on regularisation by denoising (RED) and we validate that the graph signal denoiser satisfies the conditions for the application of RED.
- Signal denoising accuracy improves in terms of RMSE.

Acknowledgement

This work is supported in part by JSPS KAKENHI (23K26110).

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